Eur. Phys. J. C 52, 315–323 (2007)
DOI 10.1140/epjc/s10052-007-0372-x

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Tensor meson exchange at low energies

G. Ecker[a], C. Zauner

Faculty of Physics, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria

Received: 16 May 2007 / Revised version: 29 June 2007 /
Published online: 1 August 2007 –

Abstract. We complete the analysis of meson resonance contributions to chiral low-energy constants of $O(p^4)$ by including all quark–antiquark bound states with orbital angular momentum ≤ 1. Different tensor meson Lagrangians used in previous work are shown to produce the same final results for the low-energy constants, once QCD short-distance constraints are properly implemented. We also discuss the possible relevance of axial-vector mesons with odd C-parity ($J^{PC} = 1^{+-}$).

1 Introduction

Chiral perturbation theory (CHPT) [1–3] is the effective field theory of the Standard Model at low energies. Its explicit degrees of freedom are the pseudoscalar mesons, the pseudo-Goldstone bosons of spontaneous chiral symmetry breaking. Since CHPT is to describe all manifestations of the Standard Model at low energies, heavier degrees of freedom must be present in the theory as well. As in all effective theories, heavy states manifest themselves in the coupling constants of the effective theory called low-energy constants (LECs) in CHPT. Realistic estimates of chiral LECs are essential for the predictive power of CHPT.

In the strong mesonic sector, both empirical and theoretical evidence suggests that chiral LECs are saturated by the lowest-lying meson resonances. In particular, the LECs of next-to-leading order, $O(p^4)$, are dominated by vector and axial-vector meson exchange [4–6], and to a lesser extent by scalar and pseudoscalar exchange (see [7, 8] for recent reviews). A systematic framework for incorporating meson resonance exchange is based on the $1/N_c$ expansion of QCD [9–15]. Although large N_c predicts an infinite number of mesons (stable to leading order in $1/N_c$), it is clear that the lowest-lying states will be most important. In fact, meson resonance exchange contributions to the LECs of $O(p^4)$ scale as $c_{\mathrm{R}}/M_{\mathrm{R}}^2$ for a resonance with mass M_{R}, where c_{R} is a measure of resonance couplings. Both the strong coupling to pseudoscalars and the comparatively low masses of the lightest vector meson nonet are responsible for the success of vector meson dominance. The relevance of other multiplets must be investigated case by case.

Of all $\overline{q}q$ bound states with orbital angular momentum $L \leq 1$, only the states with $J^{PC} = 2^{++}$ (tensor mesons) and $J^{PC} = 1^{+-}$ (axial-vector mesons with odd C-parity) still need to be analyzed. Although tensor meson contributions to chiral LECs were already considered by Donoghue et al. [6] nearly 20 years ago, very different predictions can be found in the literature [16–22]. On the other hand, the influence of 1^{+-} resonances on chiral LECs has not been considered previously. It is the purpose of the present work to settle the issue of tensor meson exchange and to investigate the possible relevance of the 1^{+-} nonet at low energies. We work in the framework of chiral SU(3) but compare also with previous predictions for tensor contributions within chiral SU(2).

In Sect. 2 we recall the phenomenological status of the $O(p^4)$ LECs L_1, L_2, L_3, L_9 and L_{10} and the evidence for resonance saturation. The importance of incorporating the proper short-distance constraints is exemplified for the vector form factor of the pion. In Sect. 3 we introduce the chiral Lagrangian for tensor mesons and the most general coupling of lowest order to the pseudoscalar mesons. It is shown that two of the three couplings do not contribute to tensor meson decay amplitudes. In the following section we investigate the constraints from axiomatic field theory for elastic meson–meson scattering. Applied to the tensor meson exchange amplitudes, the constraints fix the tensor contributions to the LECs L_1, L_2 and L_3 uniquely in terms of the single coupling constant governing tensor meson decays. We compare our results with previous work on tensor meson exchange. In Sect. 5 we turn to the axial-vector mesons with $J^{PC} = 1^{+-}$. Although they superficially contribute to the same LECs as the vector mesons, albeit with opposite sign, the same short-distance constraints that determine the vector meson contributions uniquely [5] imply the absence of all 1^{+-} contributions to the LECs of $O(p^4)$. Section 6 summarizes our conclusions. Two appendices contain basic features of the Lagrangians for symmetric (spin 2) and antisymmetric tensor fields (spin 1).

[a] e-mail: Gerhard.Ecker@univie.ac.at

2 Low-energy couplings and resonance exchange

Besides the leading-order Lagrangian

$$\mathcal{L}_2 = \frac{F^2}{4}\langle u^\mu u_\mu + \chi_+\rangle \tag{1}$$

we shall be concerned with the strong chiral Lagrangian of $O(p^4)$ [3]. For chiral SU(3), it can be written in the form

$$\begin{aligned}\mathcal{L}_4 &= L_1\langle u^\mu u_\mu\rangle^2 + L_2\langle u^\mu u^\nu\rangle\langle u_\mu u_\nu\rangle + L_3\langle(u^\mu u_\mu)^2\rangle \\ &+ L_4\langle u^\mu u_\mu\rangle\langle\chi_+\rangle + L_5\langle u^\mu u_\mu \chi_+\rangle + L_6\langle\chi_+\rangle^2 \\ &+ L_7\langle\chi_-\rangle^2 + \frac{L_8}{2}\langle\chi_+^2+\chi_-^2\rangle - \mathrm{i}L_9\langle f_{+\mu\nu}u^\mu u^\nu\rangle \\ &+ \frac{L_{10}}{4}\langle f_{+\mu\nu}f_+^{\mu\nu} - f_{-\mu\nu}f_-^{\mu\nu}\rangle \end{aligned}\tag{2}$$

in terms of the 10 LECs $L_1,\ldots,L_{10}$. The various matrix fields are defined as usual (see, e.g. [4]):

$$\begin{aligned} u_\mu &= \mathrm{i}\left\{u^\dagger(\partial_\mu - \mathrm{i}r_\mu)u - u(\partial_\mu - \mathrm{i}\ell_\mu)u^\dagger\right\}, \\ \chi_\pm &= u^\dagger\chi u^\dagger \pm u\chi^\dagger u, \\ f_{\pm\mu\nu} &= uF_{\mathrm{L}\mu\nu}u^\dagger \pm u^\dagger F_{\mathrm{R}\mu\nu}u, \\ F_{\mathrm{R}\mu\nu} &= \partial_\mu r_\nu - \partial_\nu r_\mu - \mathrm{i}[r_\mu, r_\nu], \\ F_{\mathrm{L}\mu\nu} &= \partial_\mu \ell_\nu - \partial_\nu \ell_\mu - \mathrm{i}[\ell_\mu, \ell_\nu]. \end{aligned}\tag{3}$$

Here, $u(\phi)$ is the coset space element parametrized by the Goldstone fields. The external matrix fields v_μ, a_μ, s, p are contained in $r_\mu = v_\mu + a_\mu$, $\ell_\mu = v_\mu - a_\mu$, $\chi = 2B(s+\mathrm{i}p)$. The symbol $\langle\ldots\rangle$ denotes the 3-dimensional flavor trace. The LECs of lowest order B, F are related to the quark condensate and to the pion decay constant, respectively.

Only the LECs L_1, L_2, L_3, L_9 and L_{10} will be relevant for the following analysis. The present phenomenological status and the resonance contributions of the standard variety V(1^{--}), A(1^{++}), S(0^{++}) are collected in Table 1 (P(0^{-+}) exchange does not contribute in this case). Keeping in mind that resonance exchange does not fix the renormalization scale of the renormalized LECs $L_i(\mu)$, the overall agreement with the phenomenological values suggests that V, A and S already saturate the LECs in Table 1. In fact, scalar exchange makes only a relatively small contribution to L_3. On the other hand, the situation in Table 1 certainly leaves room for additional contributions. We therefore include all meson resonances corresponding to $\overline{q}q$ bound states with orbital angular momentum $L \le 1$. From the point of view of quantum numbers, states with $J^{PC} = 2^{++}$ and 1^{+-} could in principle contribute to some of the LECs in Table 1.

We follow here the traditional approach of chiral resonance Lagrangians [4]. Compared to studying Green functions directly with a large-N_c inspired ansatz, the Lagrangian approach offers the possibility of integrating out the resonance fields once and for all in the generating functional of Green functions (to leading order in $1/N_c$), thereby generating all contributions of a given order. In addition, chiral symmetry is of course guaranteed so that chiral Ward identities are satisfied automatically.

A priori, the chiral resonance Lagrangian knows nothing about the short-distance structure of QCD. Therefore, the Lagrangian approach must always be supplemented by short-distance constraints [5]. This will turn out to be especially important for resonance contributions of the type $J^{PC} = 2^{++}$ and 1^{+-}. It will be sufficient to implement the same constraints that were used to establish the uniqueness of vector and axial-vector contributions [5] to the LECs in Table 1.

Short-distance constraints refer to Green functions or amplitudes but not to special resonance exchanges. Is it then legitimate to apply those constraints to a given resonance exchange contribution if only the sum of (an infinite number of) such exchanges must satisfy the constraints?

An instructive example is provided by the vector form factor of the pion $F_V^\pi(t)$. From the asymptotic behavior of the $I=1$ vector current two-point function in QCD we know [24] that $F_V^\pi(t)$ satisfies a dispersion relation with at most one subtraction:

$$F_V^\pi(t) = 1 + \frac{t}{\pi}\int_0^\infty \mathrm{d}t' \frac{\mathrm{Im}F_V^\pi(t')}{t'(t'-t-\mathrm{i}\epsilon)}. \tag{4}$$

To first non-trivial order in the low-energy expansion, $F_V^\pi(t)$ is given by [25]

$$F_V^\pi(t) = 1 + \frac{2}{F^2}L_9(\mu)t + \frac{2}{F^2}\Phi(t, M_\pi^2, M_K^2;\mu) + O(p^6), \tag{5}$$

where the function $\Phi(t, M_\pi^2, M_K^2;\mu)$ accounts for pion and kaon loops. The slope of the form factor gives rise to the sum rule

$$L_9(\mu) + \frac{\mathrm{d}\Phi}{\mathrm{d}t}(0, M_\pi^2, M_K^2;\mu) + O(p^6) = \frac{F^2}{2\pi}\int_0^\infty \mathrm{d}t \frac{\mathrm{Im}F_V^\pi(t)}{t^2}. \tag{6}$$

Both the scale dependence of L_9 and the loop function Φ are non-leading in $1/N_c$. Since LECs do not depend on light quark masses, we can take the chiral limit to eliminate contributions from higher-order LECs. At leading order in $1/N_c$, the absorptive part is given by

$$\mathrm{Im}F_V^\pi(t) = \frac{2\pi}{F^2}\sum_{\mathrm{R}}\kappa_{\mathrm{R}} M_{\mathrm{R}}^2 \delta(t - M_{\mathrm{R}}^2) \tag{7}$$

Table 1. Phenomenological values and theoretical estimates for the SU(3) LECs $L_i(M_\rho)$ in units of 10^{-3}. The first column shows the original values of [3], the second displays the current values taken from [23] and references therein. The third column contains the resonance saturation results of [4]. The value for L_9 was taken as input in [4]

i	[3]	[23]	[4]
1	0.7 ± 0.3	0.43 ± 0.12	0.6
2	1.3 ± 0.7	0.73 ± 0.12	1.2
3	-4.4 ± 2.5	-2.35 ± 0.37	-3.0
9	6.9 ± 0.7	5.93 ± 0.43	6.9
10	-5.5 ± 0.7	-5.09 ± 0.47	-6.0

giving rise to the form factor

$$F_{\rm V}^{\pi}(t) = 1 + \frac{2t}{F^2}\sum_{\rm R}\frac{\kappa_{\rm R}}{M_{\rm R}^2 - t - {\rm i}\epsilon}\,. \qquad (8)$$

To leading order in $1/N_c$, L_9 is therefore of the familiar form

$$L_9 = \sum_{\rm R}\frac{\kappa_{\rm R}}{M_{\rm R}^2}\,, \qquad (9)$$

where $\kappa_{\rm R}$ is related to the product of resonance couplings to the electromagnetic current and to two pions. To a given resonance we can associate unambiguously the contribution

$$L_9^{\rm R} = \frac{\kappa_{\rm R}}{M_{\rm R}^2}\,, \qquad (10)$$

even though only the total L_9 emerges from the sum rule (6). Of course, the same conclusion is reached by subjecting each resonance separately to the short-distance constraints, which in the present case are encoded in the once-subtracted dispersion relation (4).

In the approach with chiral resonance Lagrangians, consistency with short-distance constraints is not automatic. In general, local contributions from the chiral Lagrangian (2) of $O(p^4)$ must be added to achieve consistency [5]. An important lesson can be drawn from the example of the pion form factor: only pole terms in the form factor contribute to the LEC L_9. This will be of special relevance for the evaluation of 1^{+-} contributions to the LECs in Sect. 5 .

3 Tensor meson exchange

In this section we compute the effective action due to the exchange of the lowest-lying nonet of tensor mesons with $J^{PC} = 2^{++}$. We describe these particles by a symmetric Hermitian rank-2 tensor field

$$\begin{aligned} T_{\mu\nu} &= T^0_{\mu\nu}\frac{\lambda_0}{\sqrt{2}} + \frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i T^{8,i}_{\mu\nu}\,, \\ T_{\mu\nu} &= T_{\nu\mu}\,. \end{aligned} \qquad (11)$$

The octet and the singlet components are given by

$$\begin{aligned} \frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i T^{8,i} &= \begin{pmatrix} \frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} & a_2^+ & K_2^{*+} \\ a_2^- & -\frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} & K_2^{*0} \\ K_2^{*-} & \bar{K}_2^{*0} & -\frac{2f_2^8}{\sqrt{6}} \end{pmatrix}, \\ T^0 &= f_2^0\,. \end{aligned} \qquad (12)$$

The tensor nonet couples to pseudoscalar mesons via the Lagrangian (see Appendix A)

$$\mathcal{L} = -\frac{1}{2}\left\langle T_{\mu\nu} D_{\rm T}^{\mu\nu,\rho\sigma} T_{\rho\sigma}\right\rangle + \left\langle T_{\mu\nu} J_{\rm T}^{\mu\nu}\right\rangle\,, \qquad (13)$$

with a symmetric tensor current $J_{\rm T}^{\mu\nu} = J_{\rm T}^{\nu\mu}$. For the octet part, the derivatives in $D_{\rm T}^{\mu\nu,\rho\sigma}$ in (A.2) must be replaced by chirally covariant derivatives. This modification will not affect the structure of the effective action up to $O(p^4)$.

The most general symmetric tensor current $J_{\rm T}^{\mu\nu}$ of $O(p^2)$ (relevant for LECs of $O(p^4)$) consists of three terms [26, 27]:

$$\begin{aligned} J_{\rm T}^{\mu\nu} &= J_1^{\mu\nu} + g^{\mu\nu} J_2\,, \\ J_1^{\mu\nu} &= g_{\rm T}\left\{u^\mu, u^\nu\right\}\,, \quad J_2 = \beta u^\mu u_\mu + \gamma\chi_+\,. \end{aligned} \qquad (14)$$

In the following section, we will consider elastic meson–meson scattering. For this purpose, we can use the free tensor propagator (A.3) in the effective action for meson–meson scattering:

$$\begin{aligned} & S_{\rm T}^{\rm eff}(MM \to MM) \\ & = \frac{1}{2}\int {\rm d}^4x\,{\rm d}^4y\,\left\langle J_{\rm T}^{\mu\nu}(x) G^T_{\mu\nu,\rho\sigma}(x-y) J_{\rm T}^{\rho\sigma}(y)\right\rangle\,. \end{aligned} \qquad (15)$$

It will be convenient to separate the contributions of $J_1^{\mu\nu}$ and J_2 to the effective action. Due to the structure of the propagator (A.3), the effective action takes the form

$$\begin{aligned} & S_{\rm T}^{\rm eff}(MM \to MM) \\ & = \frac{1}{2}\int {\rm d}^4x\,{\rm d}^4y\,\left\langle J_1^{\mu\nu}(x) G^T_{\mu\nu,\rho\sigma}(x-y) J_1^{\rho\sigma}(y)\right\rangle \\ & \quad - \frac{1}{3M_{\rm T}^2}\int {\rm d}^4x\,\left\langle J_2(x)\left(g_{\mu\nu} - 2M_{\rm T}^{-2}\partial_\mu\partial_\nu\right) J_1^{\mu\nu}(x)\right\rangle \\ & \quad - \frac{1}{3M_{\rm T}^2}\int {\rm d}^4x\,\left\langle J_2(x)\left(2 - M_{\rm T}^{-2}\Box\right) J_2(x)\right\rangle\,. \end{aligned} \qquad (16)$$

The important observation is that the tensor current $g^{\mu\nu}J_2$ contributes only to the local part of the effective action of $O(p^4)$ and higher. As we shall see in the following section, such local actions must in fact be added to the bare tensor exchange in order to satisfy appropriate short-distance constraints. Thus, the couplings β, γ in the current J_2 can always be absorbed in the effective chiral Lagrangians. We therefore set them to zero in this section without loss of generality. Nevertheless, it will turn out to be convenient to reinstall $\beta \neq 0$ for the discussion of short-distance constraints in pion–pion scattering in the next section.

The couplings β and γ are arbitrary because, in contrast to the coupling constant $g_{\rm T}$ in (14), they cannot be determined from the partial decay widths of tensor resonances. The tensor field is traceless on-shell ($\varepsilon^\mu_{\ \mu} = 0$ in (A.8)) so that β and γ do not enter matrix elements for tensor meson decays.

In order to determine the LECs of $O(p^4)$ due to tensor exchange, we need to take the leading term of the propagator (A.3) in an expansion in $1/M_{\rm T}^2$:

$$\begin{aligned} & G^T_{\mu\nu,\rho\sigma}(x)\big|_{O(M_{\rm T}^{-2})} \\ & = \frac{1}{6M_{\rm T}^2}\left\{3\left(g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}\right) - 2g_{\mu\nu}g_{\rho\sigma}\right\}\delta^{(4)}(x)\,. \end{aligned} \qquad (17)$$

From the first term in the effective action (16) we then obtain an effective Lagrangian $\mathcal{L}^T_{4,\mathrm{bare}}$ of $O(p^4)$ from tensor exchange:

$$\mathcal{L}^T_{4,\mathrm{bare}} = \frac{g_\mathrm{T}^2}{2M_\mathrm{T}^2}\left\{\langle u_\mu u^\mu\rangle^2 + 2\,\langle u_\mu u_\nu\rangle\,\langle u^\mu u^\nu\rangle - \frac{10}{3}\,\langle u^\mu u_\mu u^\nu u_\nu\rangle\right\}. \tag{18}$$

Comparing with the general Lagrangian (2) of $O(p^4)$, we find the following (bare) LECs due to tensor exchange:

$$L^T_{1,\mathrm{bare}} = \frac{g_\mathrm{T}^2}{2M_\mathrm{T}^2}\,,\quad L^T_{2,\mathrm{bare}} = 2L^T_{1,\mathrm{bare}}\,,$$
$$L^T_{3,\mathrm{bare}} = -\frac{5g_\mathrm{T}^2}{3M_\mathrm{T}^2}\,. \tag{19}$$

We denote the Lagrangian (18) and the LECs (19) as bare quantities, because we still have to check for consistency with the short-distance structure of QCD. As we shall see in the next section, the short-distance constraints will modify the bare LECs substantially. Finally, we note that exchange of the tensor nonet is of course compatible with the large-N_c prediction $L_2 = 2L_1$.

4 Short-distance constraints for tensor exchange

The LECs L_1, L_2, L_3 all contribute to meson–meson scattering. It will turn out to be sufficient to investigate forward dispersion relations for elastic meson–meson scattering amplitudes. Since the L_i do not depend on light quark masses, all calculations will be performed in the chiral limit.

We briefly recall the well-known structure of the forward dispersion relation for an elastic channel with $s \leftrightarrow u$ symmetry, e.g., $\pi^+\pi^0 \to \pi^+\pi^0$ (for a recent account see [8]). In this case, general quantum field theory guarantees [28, 29] that the scattering amplitude $A(\nu, t)$ satisfies a once-subtracted forward dispersion relation in $\nu = (s-u)/2$:

$$A(\nu, t=0) = A(0,0) + \frac{\nu^2}{\pi}\int_0^\infty \mathrm{d}\nu'^{\,2}\,\frac{\mathrm{Abs}\,A(\nu',0)}{\nu'^{\,2}\,(\nu^2-\nu'^{\,2})}\,. \tag{20}$$

In the chiral limit and to leading order in $1/N_c$, exchange of a resonance gives rise to an amplitude

$$A(\nu,0) = \frac{c_\mathrm{R}\nu^2}{\nu^2 - M_\mathrm{R}^4}\,, \tag{21}$$

where c_R is related to the partial decay width $\Gamma(\mathrm{R}\to MM)$ in this case. On the other hand, resonance exchange from a chiral resonance Lagrangian such as (13) will produce an amplitude of the general form

$$A_\mathrm{R}(\nu,0) = \frac{P_\mathrm{R}(\nu^2)}{\nu^2 - M_\mathrm{R}^4}\,, \tag{22}$$

with a polynomial $P_\mathrm{R}(\nu^2)$ satisfying the on-shell condition $P_\mathrm{R}(M_\mathrm{R}^4) = c_\mathrm{R} M_\mathrm{R}^4$. Decomposing the polynomial $P_\mathrm{R}(\nu^2)$ as

$$P_\mathrm{R}(\nu^2) = P_\mathrm{R}(M_\mathrm{R}^4) + \left(\nu^2 - M_\mathrm{R}^4\right)\overline{P_\mathrm{R}}(\nu^2)\,, \tag{23}$$

the equality $A_\mathrm{R}(\nu,0) = A(\nu,0)$ forces $\overline{P_\mathrm{R}}(\nu^2)$ to be a constant,

$$\overline{P_\mathrm{R}}(\nu^2) = c_\mathrm{R}\,. \tag{24}$$

This will not be the case for our tensor meson Lagrangian (13). Therefore, the dispersion relation (20) requires the addition of a (counterterm) polynomial $P_\mathrm{c}(\nu^2)$ from the effective chiral Lagrangians of $O(p^4)$ and higher:

$$A_\mathrm{R}(\nu,0) = P_\mathrm{c}(\nu^2) + \overline{P_\mathrm{R}}(\nu^2) + \frac{P_\mathrm{R}(M_\mathrm{R}^4)}{\nu^2 - M_\mathrm{R}^4}\,. \tag{25}$$

The counterterm polynomial $P_\mathrm{c}(\nu^2)$ is then fixed by the short-distance constraint to satisfy

$$P_\mathrm{c}(\nu^2) + \overline{P_\mathrm{R}}(\nu^2) = c_\mathrm{R}\,, \tag{26}$$

ensuring at the same time the correct low-energy behavior of the resonance exchange amplitude:

$$A_\mathrm{R}(\nu,0) = A(\nu,0) = -\frac{c_\mathrm{R}}{M_\mathrm{R}^4}\nu^2 + O(p^8)\,. \tag{27}$$

The coefficient of ν^2 depends only on the mass and on the partial decay width of the resonance and it defines the resonance contribution to a certain combination of the L_i.

Even though we are interested in the low-energy behavior, the same conclusion is obtained by comparing the high-energy behavior of $A_\mathrm{R}(\nu,0)$ with $A(\nu,0)$:

$$\lim_{\nu^2\to\infty} A(\nu,0) = c_\mathrm{R} = \lim_{\nu^2\to\infty} A_\mathrm{R}(\nu,0)$$
$$= \lim_{\nu^2\to\infty}\left(P_\mathrm{c}(\nu^2) + \overline{P_\mathrm{R}}(\nu^2)\right)\,. \tag{28}$$

It will often be more convenient to investigate the high-energy behavior.

4.1 Elastic meson–meson scattering

The meson–meson scattering amplitude due to tensor meson exchange can be extracted from the effective action (16). Following the discussion in Sect. 3, we are led to include only the interaction term $J_1^{\mu\nu}$. It turns out that the short-distance constraints embodied in the forward dispersion relation (20) would then require the addition of local terms not only of $O(p^4)$ but also of $O(p^6)$.

By a judicious choice of the (a priori) arbitrary coupling constant β in (14) we can avoid having to include terms of $O(p^6)$ at this stage, where we are only interested in the LECs of $O(p^4)$. The specific value of β ensuring the absence of p^6 terms is

$$\beta = -g_\mathrm{T} \tag{29}$$

corresponding to a special structure of the tensor coupling $J_{\rm T}^{\mu\nu}$. In this case, the bilinear terms in u_μ in $J_{\rm T}^{\mu\nu}$ occur in the same combination as in the energy-momentum tensor [30] associated with the lowest-order chiral Lagrangian (1). It was already observed by Bellucci et al. [26, 27] that this choice of $J_{\rm T}^{\mu\nu}$ leads to a smoother high-energy behavior than in the general case.

We hasten to emphasize that our final values for the LECs L_1, L_2, L_3 will be completely independent of the choice of β. As already pointed out, β appears in the scattering amplitude only through polynomial terms that can always be absorbed in contributions from the chiral Lagrangians of $O(p^4)$ (and $O(p^6)$ in general). The main advantage of the choice (29) is that it allows us to omit the qualifying statement "up to terms of $O(p^6)$" after every other equation.

We now consider pion–pion scattering. The scattering amplitude $T(\pi_a\pi_b \to \pi_c\pi_d) \equiv T_{ab,cd}(s,t,u)$ can be expressed in terms of a single function $A(s,t,u) = A(s,u,t)$ as

$$T_{ab,cd}(s,t,u) = A(s,t,u)\delta_{ab}\delta_{cd} + A(t,s,u)\delta_{ac}\delta_{bd} + A(u,t,s)\delta_{ad}\delta_{bc}\,. \tag{30}$$

From the effective action (16) we obtain the tensor exchange amplitude in the chiral limit (for $\beta = -g_{\rm T}$):

$$A_{\rm T}(s,t,u) = \frac{2g_{\rm T}^2}{F^4(M_{\rm T}^2 - s)}\left[(t-u)^2 - \frac{s^2}{3}\right]. \tag{31}$$

In order to satisfy the short-distance constraints, we have to add an explicit local amplitude from the $O(p^4)$ Lagrangian (2):

$$A_{\rm SD}(s,t,u) = \frac{4}{F^4}\left[\left(2L_1^{\rm SD} + L_3^{\rm SD}\right)s^2 + L_2^{\rm SD}(t^2+u^2)\right]. \tag{32}$$

The $L_i^{\rm SD}$ will be determined from the short-distance constraints but they are of course not the final values of the LECs associated with tensor meson exchange. The final values are obtained by expanding the complete amplitude $A_{\rm T}(s,t,u) + A_{\rm SD}(s,t,u)$ to $O(p^4)$:

$$\begin{aligned} &A_{\rm T}(s,t,u) + A_{\rm SD}(s,t,u) \\ &= \frac{2g_{\rm T}^2}{F^4 M_{\rm T}^2}\left[(t-u)^2 - \frac{s^2}{3}\right] \\ &\quad + \frac{4}{F^4}\left[\left(2L_1^{\rm SD} + L_3^{\rm SD}\right)s^2 + L_2^{\rm SD}(t^2+u^2)\right] + O(p^6) \\ &= \frac{4}{F^4}\left[s^2\left(2L_1^{\rm SD} + L_3^{\rm SD} - \frac{2g_{\rm T}^2}{3M_{\rm T}^2}\right)\right. \\ &\quad \left. + (t^2+u^2)\left(L_2^{\rm SD} + \frac{g_{\rm T}^2}{M_{\rm T}^2}\right)\right] + O(p^6)\,. \end{aligned} \tag{33}$$

We can immediately read off the total tensor exchange contributions $2L_1^T + L_3^T$ and L_2^T from the last expansion. To obtain L_1^T and L_3^T separately, we either need another independent channel, e.g., elastic $K\pi$ scattering, or we appeal to the large-N_c relation $L_2 = 2L_1$ that is of course respected by exchange of a tensor nonet. Both approaches lead to the same results:

$$\begin{aligned} L_2^T &= 2L_1^T = \frac{g_{\rm T}^2}{M_{\rm T}^2} + L_2^{\rm SD}\,, \\ L_3^T &= -\frac{5g_{\rm T}^2}{3M_{\rm T}^2} + L_3^{\rm SD}\,. \end{aligned} \tag{34}$$

Referring back to (19), we observe that the bare tensor contributions to the L_i are identical. This equality is to some extent accidental because it happens to hold specifically for the special cases $\beta = 0$ (adopted in Sect. 3) and $\beta = -g_{\rm T}$ assumed here. For other values of β the bare term $L_{3,\rm bare}^T$ will in general be different, while $L_{1,\rm bare}^T$, $L_{2,\rm bare}^T$ remain unchanged [31]. However, as the following arguments will show, the total values L_i^T will always be the same.

In order to determine the short-distance induced contributions $L_i^{\rm SD}$, we consider the following two channels with $s \leftrightarrow u$ symmetry:

$$A\,(\pi^+\pi^0 \to \pi^+\pi^0) = A(t,s,u) \tag{35}$$

$$A\,(\pi^0\pi^0 \to \pi^0\pi^0) = A(s,t,u) + A(t,s,u) + A(u,t,s)\,. \tag{36}$$

The forward scattering amplitude for the $\pi^+\pi^0$ channel is therefore

$$A_{\rm T}(\nu,0)|_{\pi^+\pi^0\to\pi^+\pi^0} = \frac{8g_{\rm T}^2}{F^4 M_{\rm T}^2}\nu^2 + \frac{8}{F^4}L_2^{\rm SD}\nu^2\,. \tag{37}$$

From the general discussion of the forward amplitude at the beginning of this section we conclude that $A_{\rm T}(\nu,0)$ must vanish for the case of $\pi^+\pi^0$ scattering (absence of a pole term). This constraint fixes $L_2^{\rm SD}$ to be

$$L_2^{\rm SD} = -\frac{g_{\rm T}^2}{M_{\rm T}^2}\,. \tag{38}$$

For the second channel we find

$$\begin{aligned} A_{\rm T}(\nu,0)|_{\pi^0\pi^0\to\pi^0\pi^0} &= \frac{8g_{\rm T}^2 M_{\rm T}^2}{3F^4}\frac{\nu^2}{M_{\rm T}^4 - \nu^2} + \frac{8g_{\rm T}^2}{F^4 M_{\rm T}^2}\nu^2 \\ &\quad + \frac{8}{F^4}\left(2L_1^{\rm SD} + 2L_2^{\rm SD} + L_3^{\rm SD}\right)\nu^2\,. \end{aligned} \tag{39}$$

Together with (38), the structure of the forward dispersion relation requires

$$2L_1^{\rm SD} + L_3^{\rm SD} = \frac{g_{\rm T}^2}{M_{\rm T}^2}\,. \tag{40}$$

As before, we can either appeal to large N_c or investigate additional meson–meson scattering channels to arrive at the following results for the $L_i^{\rm SD}$:

$$L_2^{\rm SD} = 2L_1^{\rm SD} = -\frac{g_{\rm T}^2}{M_{\rm T}^2}\,, \quad L_3^{\rm SD} = \frac{2g_{\rm T}^2}{M_{\rm T}^2}\,. \tag{41}$$

In fact, all channels are compatible with these values. Inserting into (34), we obtain the complete LECs L_i^T due to tensor meson exchange:

$$L_1^T = L_2^T = 0\,, \quad L_3^T = \frac{g_{\mathrm{T}}^2}{3M_{\mathrm{T}}^2}\,. \tag{42}$$

Comparing with the bare LECs (19), we observe that the short-distance constraints have eliminated L_1 and L_2 altogether. Moreover, the absolute magnitude of L_3 is reduced by a factor of five. Once again, we stress that the so-called bare values (19) have no intrinsic meaning. Only the final values (42) can be associated with tensor meson exchange.

4.2 Numerical discussion and comparison with previous work

The tensor coupling constant g_{T} defined in (14) can be determined from the decay rate $\Gamma(f_2(1270) \to \pi\pi)$. To a good approximation (see, e.g. [32]), the $f_2(1270)$ is the non-strange partner of an ideal mixture of the SU(3) singlet and octet isosinglet states:

$$f_2(1270)_{\mu\nu} = \left(\sqrt{2}T^0_{\mu\nu} + T^{8,8}_{\mu\nu}\right)/\sqrt{3}\,. \tag{43}$$

To the accuracy needed for our purposes, the assumption of ideal mixing is completely sufficient.

The decay rate $\Gamma(f_2(1270) \to \pi\pi)$ is then given by

$$\Gamma(f_2(1270) \to \pi\pi) = \frac{g_{\mathrm{T}}^2 M_{\mathrm{T}}^3}{40\pi F_\pi^4}\left(1 - 4M_\pi^2/M_{\mathrm{T}}^2\right)^{5/2}\,. \tag{44}$$

With $M_{\mathrm{T}} = M(f_2(1270))$ and $\Gamma(f_2(1270) \to \pi\pi)$ taken from PDG 2006 [33] and with $F_\pi = 92.4\,\mathrm{MeV}$, one finds

$$|g_{\mathrm{T}}| = 28\,\mathrm{MeV}\,. \tag{45}$$

This value should be compared with the corresponding vector and scalar couplings G_{V} [5] and c_d [15, 34–36]:

$$|G_{\mathrm{V}}| \simeq \frac{F_\pi}{\sqrt{2}} = 65\,\mathrm{MeV},$$
$$46\,\mathrm{MeV} = \frac{F_\pi}{2} \lesssim |c_d| \lesssim \frac{F_\pi}{\sqrt{2}}\,. \tag{46}$$

Thus, the tensor coupling to pions is not much smaller than the corresponding vector and scalar couplings. Nevertheless, the only non-zero contribution of tensor exchange to the LECs of $O(p^4)$,

$$L_3^T = \frac{g_{\mathrm{T}}^2}{3M_{\mathrm{T}}^2} = 0.16 \times 10^{-3}\,, \tag{47}$$

is considerably smaller than the sum of vector and scalar contributions. This is only partly due to the larger mass M_{T}. We recall that the so-called bare value in (19),

$$L_{3,\mathrm{bare}}^T = -\frac{5g_{\mathrm{T}}^2}{3M_{\mathrm{T}}^2} = -0.80 \times 10^{-3}\,, \tag{48}$$

would amount to a non-negligible contribution to L_3 (see Table 1).

Table 2. Tensor contributions to the SU(2) LECs l_1, l_2 from various sources

	$l_1^T \times 10^3$	$l_2^T \times 10^3$
Donoghue, Ramirez, Valencia [6]	−0.6	1.9
Dobado, Pelaez [20]	−0.6	1.9
Suzuki [16]	−0.5	2.0
Katz, Lewandowski, Schwartz [21, 22]	−0.7	2.1
Toublan [17, 18]	0.3	0
Ananthanarayan [19]	0.3	0
this work	0.3	0

In much of the previous literature, tensor meson exchange was considered in the framework of chiral SU(2). To $O(p^4)$, the SU(3) results can be translated to the SU(2) LECs l_i^T through the relations [3]

$$l_1^T = 4L_1^T + 2L_3^T\,, \tag{49}$$
$$l_2^T = 4L_2^T\,. \tag{50}$$

The numerical values for l_1^T and l_2^T from different sources are collected in Table 2. As far as we are aware, the first determination of tensor contributions to the l_i was performed by Donoghue et al. [6]. Their results are identical to those in [20] and they correspond exactly to our bare LECs in (19) ($\beta = 0$ in our notation). Different tensor meson couplings were used in [16, 21, 22]. In the Lagrangian of [21, 22], the $f_2(1270)$ is assumed to couple like the graviton to the energy-momentum tensor ($\beta = -g_{\mathrm{T}}$). Of all the previous work, only Toublan [17, 18] and Ananthanarayan [19] took short-distance constraints into account. In [19] different versions of dispersion relations for $\pi\pi$ scattering[1] were analyzed to determine the f_2 contribution to the l_i. Although Toublan used a different Lagrangian for the tensor fields and applied slightly different short-distance arguments, we agree with his results in the SU(2) limit. The agreement with [17–19] underscores our claim that the final results for tensor meson exchange to the LECs are model independent, once the high-energy conditions are properly implemented. On the other hand, the results in Table 2 document rather convincingly that the high-energy constraints are essential to arrive at unique values for the contributions of tensor meson exchange.

5 1^{+-} resonances

The contributions of axial-vector mesons with odd C-parity ($J^{PC} = 1^{+-}$) to the LECs of $O(p^4)$ have not been considered up to now. This may partly be due to the fact that the corresponding nonet has not been unambiguously identified yet [33]. Only the states $h_1(1170)$ and $b_1(1235)$ are

[1] Starting at $O(p^6)$, crossing symmetry imposes additional constraints on resonance exchange contributions with spin ≥ 2 [19, 37].

listed in the PDG booklet as respectable resonances. Moreover, there is only limited information on decay widths and branching ratios. Nevertheless, there are good arguments for the existence of a complete nonet (e.g. [32]). Although the masses of this nonet are considerably larger than those of the lowest-lying vector mesons, they are comparable with the masses of the axial-vector mesons with positive C-parity ($J^{PC} = 1^{++}$). Since the latter make an important contribution to L_{10} [4], there is a priori no reason to disregard the 1^{+-} nonet.

To investigate contributions of spin-1 exchange to the LECs of $O(p^4)$, it is convenient to describe those mesons in terms of antisymmetric tensor fields (see Appendix B). Denoting the nonet spin-1 field as $H_{\mu\nu}$, the kinetic Lagrangian is given by

$$\mathcal{L}_H = \left\langle -\frac{1}{2}\nabla^\mu H_{\mu\nu}\nabla_\rho H^{\rho\nu} + \frac{M_H^2}{4} H_{\mu\nu}H^{\mu\nu} \right\rangle . \tag{51}$$

As in the case of 1^{--} and 1^{++} exchange [4], the SU(3) singlet cannot contribute at $O(p^4)$. Under parity and charge conjugation, the relevant octet field $H_{\mu\nu}$ transforms as

$$\begin{aligned} H_{\mu\nu}(t,\mathbf{x}) &\xrightarrow{P} -\epsilon(\mu)\epsilon(\nu)H_{\mu\nu}(t,-\mathbf{x}) , \\ H_{\mu\nu}(x) &\xrightarrow{C} -H_{\mu\nu}^T(x) . \end{aligned} \tag{52}$$

The most general chiral invariant interaction of $O(p^2)$ of the 1^{+-} mesons with the Goldstone bosons respecting P and C invariance is then given by

$$\mathcal{L}_{\rm int}[H(1^{+-})] = \langle H_{\mu\nu} J_H^{\mu\nu}\rangle \tag{53}$$

with an antisymmetric tensor current

$$J_H^{\mu\nu} = \frac{F_H}{4\sqrt{2}}\varepsilon^{\mu\nu\rho\sigma} f_{+\rho\sigma} + \frac{\mathrm{i}G_H}{2\sqrt{2}}\varepsilon^{\mu\nu\rho\sigma}u_\rho u_\sigma. \tag{54}$$

As already pointed out, the SU(3) singlet field does not couple because of

$$\langle J_H^{\mu\nu}\rangle = 0. \tag{55}$$

Expanding around the classical solution in the usual manner, we obtain the effective action induced by 1^{+-} exchange

$$S_H^{\rm eff} = \frac{1}{2}\int \mathrm{d}^4x \,\langle H_{\mu\nu}^{\rm cl} J_H^{\mu\nu}\rangle \, . \tag{56}$$

To $O(p^4)$, the effective action is

$$S_H^{\rm eff} = \int \mathrm{d}^4x \mathcal{L}_{4,\rm bare}^H(x) , \tag{57}$$

with the Lagrangian $\mathcal{L}_{4,\rm bare}^H$ given by

$$\mathcal{L}_{4,\rm bare}^H = -\frac{1}{M_H^2}\langle J_{H\mu\nu} J_H^{\mu\nu}\rangle \, . \tag{58}$$

A straightforward calculation produces an effective Lagrangian of the form (2) with

$$L_{1,\rm bare}^H = \frac{1}{2}L_{2,\rm bare}^H = -\frac{G_H^2}{8M_H^2} , \quad L_{3,\rm bare}^H = -6L_{1,\rm bare}^H ,$$

$$L_{9,\rm bare}^H = -\frac{F_H G_H}{2M_H^2} , \quad L_{10,\rm bare}^H = \frac{F_H^2}{4M_H^2} \, . \tag{59}$$

We have chosen the normalization of couplings in the current (54) to facilitate comparison with vector meson exchange. Comparing with the results of [4], one finds that the replacements $F_{\rm V} \to F_H$ and $G_{\rm V} \to G_H$ yield the LECs in (59) except for an overall change of sign. Except possibly for L_9 (we do not know the relative sign of F_H, G_H in contrast to $F_{\rm V}G_{\rm V} > 0$), these results seem to suggest that exchange of 1^{+-} resonances reduces the effect of vector meson exchange. The relevant question is then: by how much?

The bare LECs $L_{i,\rm bare}^H$ $(i = 1,2,3)$ contribute to elastic meson–meson scattering. In analogy to the case of vector mesons [4], we are led to determine the coupling constant G_H from the decays of 1^{+-} resonances to two pseudoscalar mesons. But parity conservation does not allow for such decays. Consequently, H exchange can only lead to a polynomial contribution to the elastic meson–meson amplitude. For example, the pion–pion scattering amplitude is given by

$$A_H(s,t,u) = \frac{G_H^2}{M_H^2 F^4}(2s^2 - t^2 - u^2), \tag{60}$$

in accordance with (59). But this form of the amplitude is not compatible with the structure of the dispersion relations discussed in Sect. 4. Therefore, the short-distance constraints require the introduction of an additional contribution from the $O(p^4)$ Lagrangian (2) that completely cancels the H exchange contribution (60):

$$A_{\rm SD}(s,t,u) = -A_H(s,t,u) \, . \tag{61}$$

Similarly, H exchange contributes a term linear in t to the vector form factor of the pion:

$$F_{\rm V}^H(t) = -\frac{F_H G_H}{M_H^2 F^2} t \, . \tag{62}$$

Again, there is no pole term because H mesons cannot decay into two pions. As discussed in Sect. 2, the absence of a pole contribution implies that H exchange does not contribute to L_9.

Finally, we turn to the VV–AA two-point function

$$\begin{aligned} &\mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}px}\langle 0|T\left[V_\mu^i(x)V_\nu^j(0) - A_\mu^i(x)A_\nu^j(0)\right]|0\rangle \\ &\quad = \delta_{ij}\left[\left(p_\mu p_\nu - g_{\mu\nu}p^2\right)\Pi_{\rm LR}^{(1)}(p^2) + p_\mu p_\nu \Pi_{\rm LR}^{(0)}(p^2)\right] . \end{aligned} \tag{63}$$

According to QCD the invariant function $\Pi_{\rm LR}^{(1)}(p^2)$ satisfies an unsubtracted dispersion relation [24]. Again, H exchange is incompatible with the short-distance constraint because it produces a constant contribution corresponding to $L_{10,\rm bare}^H$ in (59). This contribution must again be cancelled by a local counterterm leading to the final conclusion that there are no 1^{+-} exchange contributions to the LECs of $O(p^4)$ at all:

$$L_i^H = 0 \quad (i = 1,\ldots,10) \, . \tag{64}$$

6 Conclusions

The saturation of low-energy constants of $O(p^4)$ by the exchange of V, A, S and P meson resonances is a generally accepted feature of strong dynamics at low energies. Chiral vector meson dominance can easily be understood because of the strong coupling to the pseudoscalars and the comparatively low masses of the lowest-lying vector meson nonet. On the other hand, it is much less obvious why A, S and P resonances should be more important than the 2^{++} and 1^{+-} states with similar masses. The latter two multiplets complete the spectrum of $\overline{q}q$ bound states with orbital angular momentum ≤ 1.

Setting up the most general chiral Lagrangians for 2^{++} and 1^{+-} fields and integrating out the resonance fields, the tensor meson contributions to the LECs L_1, L_2 and L_3 seem to depend on a coupling that cannot be determined from tensor meson decays. In the case of 1^{+-} exchange, the same LECs are affected that receive vector meson contributions, albeit with opposite sign. Both results are superficial and must be confronted with the short-distance constraints of QCD.

In the tensor meson case, the constraints of axiomatic field theory for elastic meson–meson scattering are actually sufficient to show that only L_3 receives a non-zero contribution. The resulting value $L_3^T = 0.16 \times 10^{-3}$ is completely negligible compared to the sum of vector and scalar contributions. Our results agree with those in [17–19] in the limit of chiral SU(2), but we disagree with all other predictions in the literature.

The final results for 1^{+-} exchange are even more pronounced. The combined short-distance constraints for elastic meson–meson scattering, the vector form factor of the pion and the VV–AA two-point function eliminate all contributions of 1^{+-} exchange to the LECs of $O(p^4)$.

The final conclusion can be summarized in one sentence: the dominance of V, A, S, P exchange contributions to the LECs of $O(p^4)$ is not an accident.

Acknowledgements. We are particularly grateful to Jürg Gasser for many helpful discussions and for providing us with his notes on symmetric tensor fields. We also thank Balasubramanian Ananthanarayan, Roland Kaiser, Heiri Leutwyler, Helmut Neufeld, Jorge Portolés and Matthew Schwartz for clarifying comments.

Appendix A: Symmetric tensor fields for spin 2

The Lagrangian for a Hermitian spin-2 field $T_{\mu\nu}$ coupled linearly to a source $J_{\mu\nu}$ can be written in the form [26, 27, 38]

$$\mathcal{L} = -\frac{1}{2} T_{\mu\nu} D_{\mathrm{T}}^{\mu\nu,\rho\sigma} T_{\rho\sigma} + T_{\mu\nu} J^{\mu\nu} , \tag{A.1}$$

with $T_{\mu\nu} = T_{\nu\mu}$, $J_{\mu\nu} = J_{\nu\mu}$ and

$$\begin{aligned} D_{\mathrm{T}}^{\mu\nu,\rho\sigma} &= (\Box + M_{\mathrm{T}}^2) \left[\frac{1}{2} (g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) - g^{\mu\nu} g^{\rho\sigma} \right] \\ &\quad + g^{\rho\sigma} \partial^\mu \partial^\nu + g^{\mu\nu} \partial^\rho \partial^\sigma \\ &\quad - \frac{1}{2} (g^{\nu\sigma} \partial^\mu \partial^\rho + g^{\rho\nu} \partial^\mu \partial^\sigma + g^{\mu\sigma} \partial^\rho \partial^\nu + g^{\rho\mu} \partial^\sigma \partial^\nu) . \end{aligned} \tag{A.2}$$

The Feynman propagator is given by

$$\begin{aligned} G_{\mu\nu,\rho\sigma}^T(x) &= \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{e}^{-\mathrm{i}kx} P_{\mu\nu,\rho\sigma}(k)}{M_{\mathrm{T}}^2 - k^2 - \mathrm{i}\epsilon} \\ P_{\mu\nu,\rho\sigma} &= \frac{1}{2} (P_{\mu\rho} P_{\nu\sigma} + P_{\nu\rho} P_{\mu\sigma}) - \frac{1}{3} P_{\mu\nu} P_{\rho\sigma} \\ P_{\mu\nu} &= g^{\mu\nu} - \frac{k_\mu k_\nu}{M_{\mathrm{T}}^2} , \end{aligned} \tag{A.3}$$

satisfying the differential equation

$$D_{\mathrm{T}}^{\mu\nu,\lambda\rho} G_{\lambda\rho,\sigma\tau}^T(x) = \frac{1}{2} (\delta_\sigma^\mu \delta_\tau^\nu + \delta_\sigma^\nu \delta_\tau^\mu) \, \delta^{(4)}(x) . \tag{A.4}$$

The classical equation of motion

$$D_{\mathrm{T}}^{\mu\nu,\rho\sigma} T_{\rho\sigma} = J^{\mu\nu} \tag{A.5}$$

has the solution

$$T_{\mu\nu}^{\mathrm{cl}}(x) = \int \mathrm{d}^4 y G_{\mu\nu,\rho\sigma}^T(x-y) J^{\rho\sigma}(y) . \tag{A.6}$$

Without the inclusion of auxiliary fields in the Lagrangian [17, 18, 39], the tensor field $T_{\mu\nu}$ is neither traceless nor transverse. However, the corresponding components do not propagate in accordance with the spin-2 nature of the field:

$$\begin{aligned} P^\mu_{\mu,\rho\sigma}(k) &= \frac{k^2 - M_{\mathrm{T}}^2}{3M_{\mathrm{T}}^2} \left(g_{\rho\sigma} + \frac{2k_\rho k_\sigma}{M_{\mathrm{T}}^2} \right) , \\ k^\mu P_{\mu\nu,\rho\sigma}(k) &= \frac{M_{\mathrm{T}}^2 - k^2}{6M_{\mathrm{T}}^2} \left(3k_\rho P_{\nu\sigma} + 3k_\sigma P_{\nu\rho} - 2k_\nu P_{\rho\sigma} \right) . \end{aligned} \tag{A.7}$$

The one-particle matrix element for a spin-2 particle with momentum k and polarization λ is expressed in terms of the polarization tensor $\varepsilon_{\mu\nu}(k;\lambda)$:

$$\begin{aligned} &\langle 0 \,| T_{\mu\nu}(0) |\, T(k;\lambda) \rangle = \varepsilon_{\mu\nu}(k;\lambda) \\ &\varepsilon_{\mu\nu} = \varepsilon_{\nu\mu} , \quad k^\mu \varepsilon_{\mu\nu} = 0 , \quad \varepsilon^\mu_\mu = 0 . \end{aligned} \tag{A.8}$$

The explicit form of the polarization tensor can be found, e.g., in [40]. For the decay rate of an unpolarized spin-2 particle one needs the sum over polarizations

$$\sum_\lambda \varepsilon_{\mu\nu}(k;\lambda) \varepsilon_{\rho\sigma}(k;\lambda)^* = P_{\mu\nu,\rho\sigma}(k) , \tag{A.9}$$

where $P_{\mu\nu,\rho\sigma}(k)$ is defined in (A.3).

Appendix B: Antisymmetric tensor fields for spin 1

For completeness, we collect in this appendix a few basic formulas for the description of spin-1 fields in terms of antisymmetric tensor fields.

The Lagrangian for a Hermitian spin-1 field $H_{\mu\nu}$ coupled linearly to a source $J_{\mu\nu}$ can be written in the form (e.g., Appendix A in [4])

$$\mathcal{L} = \frac{1}{2} H_{\mu\nu} D_H^{\mu\nu,\rho\sigma} H_{\rho\sigma} + H_{\mu\nu} J^{\mu\nu} \,, \tag{B.1}$$

with $H_{\mu\nu} = -H_{\nu\mu}$, $J_{\mu\nu} = -J_{\nu\mu}$ and

$$\begin{aligned} D_H^{\mu\nu,\rho\sigma} &= \frac{1}{4}\partial_\lambda \left[g^{\rho\lambda} \left(\partial^\mu g^{\nu\sigma} - \partial^\nu g^{\mu\sigma} \right) - g^{\sigma\lambda} \left(\partial^\mu g^{\nu\rho} - \partial^\nu g^{\mu\rho} \right) \right] \\ &\quad + \frac{M_H^2}{4} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} \right) \,. \end{aligned} \tag{B.2}$$

The Feynman propagator is given by

$$\begin{aligned} G^H_{\mu\nu,\rho\sigma}(x) &= \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{e}^{-\mathrm{i}kx} Q_{\mu\nu,\rho\sigma}(k)}{M_H^2 (M_H^2 - k^2 - \mathrm{i}\epsilon)} \,, \\ Q_{\mu\nu,\rho\sigma} &= \left[g_{\mu\rho} g_{\nu\sigma} \left(M_H^2 - k^2 \right) \right. \\ &\quad \left. + g_{\mu\rho} k_\nu k_\sigma - g_{\mu\sigma} k_\nu k_\rho - (\mu \leftrightarrow \nu) \right] \,, \end{aligned} \tag{B.3}$$

satisfying the differential equation

$$D_H^{\mu\nu,\lambda\rho} G^H_{\lambda\rho,\sigma\tau}(x) = \frac{1}{2} \left(\delta^\mu_\sigma \delta^\nu_\tau - \delta^\nu_\sigma \delta^\mu_\tau \right) \delta^{(4)}(x) \,. \tag{B.4}$$

The classical equation of motion

$$D_H^{\mu\nu,\rho\sigma} H_{\rho\sigma} = -J^{\mu\nu} \tag{B.5}$$

has the solution

$$H^{\mathrm{cl}}_{\mu\nu}(x) = -\int \mathrm{d}^4 y\, G^H_{\mu\nu,\rho\sigma}(x-y) J^{\rho\sigma}(y) \,. \tag{B.6}$$

The one-particle matrix element for a spin-1 particle with momentum k and polarization λ is expressed in terms of the usual polarization vector $\varepsilon_\mu(k;\lambda)$:

$$\begin{aligned} \langle 0 \left| H_{\mu\nu}(0) \right| H(k;\lambda) \rangle &= \mathrm{i} M_H^{-1} \left[k_\mu \varepsilon_\nu(k;\lambda) - k_\nu \varepsilon_\mu(k;\lambda) \right] \,, \\ k^\mu \varepsilon_\mu &= 0 \,. \end{aligned} \tag{B.7}$$

References

1. S. Weinberg, Physica A **96**, 327 (1979)
2. J. Gasser, H. Leutwyler, Ann. Phys. **158**, 142 (1984)
3. J. Gasser, H. Leutwyler, Nucl. Phys. B **250**, 465 (1985)
4. G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B **321**, 311 (1989)
5. G. Ecker, J. Gasser, H. Leutwyler, A. Pich, E. de Rafael, Phys. Lett. B **223**, 425 (1989)
6. J.F. Donoghue, C. Ramirez, G. Valencia, Phys. Rev. D **39**, 1947 (1989)
7. R. Kaiser, in: Large N_c QCD 2004: Proceedings of the Workshop, ed. by J.L. Goity, R.F. Lebed, A. Pich, C.L. Schat, N.N. Scoccola (World Scientific, Singapore, 2005) [arXiv:hep-ph/0502065]
8. G. Ecker, arXiv:hep-ph/0702263
9. G. 't Hooft, Nucl. Phys. B **72**, 461 (1974)
10. G. 't Hooft, Nucl. Phys. B **75**, 461 (1974)
11. E. Witten, Nucl. Phys. B **160**, 57 (1979)
12. S. Peris, M. Perrottet, E. de Rafael, JHEP **9805**, 011 (1998) [arXiv:hep-ph/9805442]
13. M. Knecht, S. Peris, M. Perrottet, E. de Rafael, Phys. Rev. Lett. **83**, 5230 (1999) [arXiv:hep-ph/9908283]
14. E. de Rafael, Nucl. Phys. Proc. Suppl. **119**, 71 (2003) [arXiv:hep-ph/0210317]
15. A. Pich, in: Phenomenology of large-N_c QCD, ed. by R.F. Lebed (World Scientific, Singapore, 2002) [arXiv:hep-ph/0205030]
16. M. Suzuki, Phys. Rev. D **47**, 1043 (1993)
17. D. Toublan, Phys. Rev. D **53**, 6602 (1996) [arXiv:hep-ph/9509217]
18. D. Toublan, Phys. Rev. D **57**, 4495 (1998) [Erratum]
19. B. Ananthanarayan, Phys. Rev. D **58**, 036 002 (1998) [arXiv:hep-ph/9802338]
20. A. Dobado, J.R. Pelaez, Phys. Rev. D **65**, 077 502 (2002) [arXiv:hep-ph/0111140]
21. E. Katz, A. Lewandowski, M.D. Schwartz, Phys. Rev. D **74**, 086 004 (2006) [arXiv:hep-ph/0510388]
22. M.D. Schwartz, private communication
23. J. Bijnens, Prog. Part. Nucl. Phys. **58**, 521 (2007) [arXiv:hep-ph/0604043]
24. E.G. Floratos, S. Narison, E. de Rafael, Nucl. Phys. B **155**, 115 (1979) and references therein
25. J. Gasser, H. Leutwyler, Nucl. Phys. B **250**, 517 (1985)
26. S. Bellucci, J. Gasser, M.E. Sainio, Nucl. Phys. B **423**, 80 (1994) [arXiv:hep-ph/9401206]
27. S. Bellucci, J. Gasser, M.E. Sainio, Nucl. Phys. B **431**, 413 (1994) [Erratum]
28. M. Froissart, Phys. Rev. **123**, 1053 (1961)
29. Y.S. Jin, A. Martin, Phys. Rev. **B 135**, 1375 (1964)
30. J.F. Donoghue, H. Leutwyler, Z. Phys. C **52**, 343 (1991)
31. C. Zauner, diploma thesis, Univ. of Vienna, 2007, (unpublished), file available from the author upon request
32. V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, JHEP **0306**, 012 (2003) [arXiv:hep-ph/0305311]
33. Particle Data Group, W.M. Yao et al., J. Phys. G **33**, 1 (2006)
34. B. Moussallam, J. Stern, arXiv:hep-ph/9404353
35. M.F.L. Golterman, S. Peris, Phys. Rev. D **61**, 034 018 (2000) [arXiv:hep-ph/9908252]
36. M. Jamin, J.A. Oller, A. Pich, Nucl. Phys. B **622**, 279 (2002) [arXiv:hep-ph/0110193]
37. B. Ananthanarayan, G. Colangelo, J. Gasser, H. Leutwyler, Phys. Rep. **353**, 207 (2001) [arXiv:hep-ph/0005297]
38. J.J. Rivers, Nuovo Cim. **34**, 5434 (1964)
39. M. Fierz, W. Pauli, Proc. R. Soc. London A **173**, 211 (1939)
40. H.M. Pilkuhn, Relativistic Particle Physics (Springer, New York, 1979)